

STRENGTH OF MATERIALS

PART I

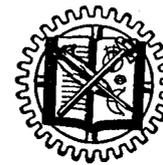
Elementary Theory and Problems

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PREFACE TO THE SECOND EDITION

In preparing the second edition of this volume, an effort has been made to adapt the book to the teaching requirements of our engineering schools.

With this in view, a portion of the material of a more advanced character which was contained in the previous edition of this volume has been removed and will be included in the new edition of the second volume. At the same time, some portions of the book, which were only briefly discussed in the first edition, have been expanded with the intention of making the book easier to read for the beginner. For this reason, chapter II, dealing with combined stresses, has been entirely rewritten. Also, the portion of the book dealing with shearing force and bending moment diagrams has been expanded, and a considerable amount of material has been added to the discussion of deflection curves by the integration method. A discussion of column theory and its application has been included in chapter VIII, since this subject is usually required in undergraduate courses of strength of materials. Several additions have been made to chapter X dealing with the application of strain energy methods to the solution of statically indetermined problems. In various parts of the book there are many new problems which may be useful for class and home work.

Several changes in the notations have been made to conform to the requirements of American Standard Symbols for Mechanics of Solid Bodies recently adopted by The American Society of Mechanical Engineers.

It is hoped that with the changes made the book will be found more satisfactory for teaching the undergraduate course of strength of materials and that it will furnish a better foundation for the study of the more advanced material discussed in the second volume.

S. TIMOSHENKO

PALO ALTO, CALIFORNIA
June 13, 1940

PREFACE TO THE FIRST EDITION

At the present time, a decided change is taking place in the attitude of designers towards the application of analytical methods in the solution of engineering problems. Design is no longer based principally upon empirical formulas. The importance of analytical methods combined with laboratory experiments in the solution of technical problems is becoming generally accepted.

Types of machines and structures are changing very rapidly, especially in the new fields of industry, and usually time does not permit the accumulation of the necessary empirical data. The size and cost of structures are constantly increasing, which consequently creates a severe demand for greater reliability in structures. The economical factor in design under the present conditions of competition is becoming of growing importance. The construction must be sufficiently strong and reliable, and yet it must be designed with the greatest possible saving in material. Under such conditions, the problem of a designer becomes extremely difficult. Reduction in weight involves an increase in working stresses, which can be safely allowed only on a basis of careful analysis of stress distribution in the structure and experimental investigation of the mechanical properties of the materials employed.

It is the aim of this book to present problems such that the student's attention will be focussed on the practical applications of the subject. If this is attained, and results, in some measure, in increased correlation between the studies of strength of materials and engineering design, an important forward step will have been made.

The book is divided into two volumes. The first volume contains principally material which is usually covered in required courses of strength of materials in our engineering

schools. The more advanced portions of the subject are of interest chiefly to graduate students and research engineers, and are incorporated in the second volume of the book. This contains also the new developments of practical importance in the field of strength of materials.

In writing the first volume of strength of materials, attention was given to simplifying all derivations as much as possible so that a student with the usual preparation in mathematics will be able to read it without difficulty. For example, in deriving the theory of the deflection curve, the *area moment method* was extensively used. In this manner, a considerable simplification was made in deriving the deflections of beams for various loading and supporting conditions. In discussing statically indeterminate systems, the *method of superposition* was applied, which proves very useful in treating such problems as continuous beams and frames. For explaining combined stresses and deriving principal stresses, use was made of the *Mohr's circle*, which represents a substantial simplification in the presentation of this portion of the theory.

Using these methods of simplifying the presentation, the author was able to condense the material and to discuss some problems of a more advanced character. For example, in discussing torsion, the twist of rectangular bars and of rolled sections, such as angles, channels, and I beams, is considered. The deformation and stress in helical springs are discussed in detail. In the theory of bending, the case of non-symmetrical cross sections is discussed, the *center of twist* is defined and explained, and the effect of shearing force on the deflection of beams is considered. The general theory of the bending of beams, the materials of which do not follow Hooke's law, is given and is applied in the bending of beams beyond the yielding point. The bending of reinforced concrete beams is given consideration. In discussing combinations of direct and bending stress, the effect of deflections on the bending moment is considered, and the limitation of the method of superposition is explained. In treating combined bending and torsion, the cases of rectangular and elliptical cross sections are dis-

cussed, and applications in the design of crankshafts are given. Considerable space in the book is devoted to methods for solving elasticity problems based on the consideration of the strain energy of elastic bodies. These methods are applied in discussing statically indeterminate systems. The stresses produced by impact are also discussed. All these problems of a more advanced character are printed in small type, and may be omitted during the first reading of the book.

The book is illustrated with a number of problems to which solutions are presented. In many cases, the problems are chosen so as to widen the field covered by the text and to illustrate the application of the theory in the solution of design problems. It is hoped that these problems will be of interest for teaching purposes, and also useful for designers.

The author takes this opportunity of thanking his friends who have assisted him by suggestions, reading of manuscript and proofs, particularly Messrs. W. M. Coates and L. H. Donnell, teachers of mathematics and mechanics in the Engineering College of the University of Michigan, and Mr. F. L. Everett of the Department of Engineering Research of the University of Michigan. He is indebted also to Mr. F. C. Wilharm for the preparation of drawings, to Mrs. E. D. Webster for the typing of the manuscript, and to the Van Nostrand Company for its care in the publication of the book.

S. TIMOSHENKO

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May 1, 1930

NOTATIONS

$\sigma_x, \sigma_y, \sigma_z$	Normal stresses on planes perpendicular to x , y and z axes.
σ_n	Normal stress on plane perpendicular to direction n .
$\sigma_{Y.P.}$	Normal stress at yield point.
σ_w	Normal working stress
τ	Shearing stress
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	Shearing stresses parallel to x , y and z axes on the planes perpendicular to y , z and x axes.
τ_w	Working stress in shear
δ	Total elongation, total deflection
ϵ	Unit elongation
$\epsilon_x, \epsilon_y, \epsilon_z$	Unit elongations in x , y and z directions
γ	Unit shear, weight per unit volume
E	Modulus of elasticity in tension and compression
G	Modulus of elasticity in shear
μ	Poisson's ratio
Δ	Volume expansion
K	Modulus of elasticity of volume
M_t	Torque
M	Bending moment in a beam
V	Shearing force in a beam
A	Cross sectional area
I_y, I_z	Moments of inertia of a plane figure with respect to y and z axes
k_y, k_z	Radii of gyration corresponding to I_y, I_z
I_p	Polar moment of inertia
Z	Section modulus
C	Torsional rigidity
l	Length of a bar, span of a beam
P, Q	Concentrated forces
t	Temperature, thickness

NOTATIONS

αCoefficient of thermal expansion, numerical coefficient

UStrain energy

wStrain energy per unit volume

hDepth of a beam, thickness of a plate

qLoad per unit length

ϕ, θAngles

pPressure

D, dDiameters

R, rRadii

WWeight, load

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